Ratio Measure

The performance measure that is in this class is billing accuracy. If a parity test were used, the sample sizes for this measure are quite large, so there is no need for a small sample technique. If one does need a small sample technique, then a re-sampling method can be used.

$$Z = \frac{\sum W Z - \sum W E(Z}{\sqrt{\sum W \ Var(Z \ |H}}$$

~~1. Calculate the aggregate test statistic, Z^T.~~ Aggregate Test Statistic (Z^T)

$$Z^T = \frac{\sum_j W_j Z_j^* - \sum_j W_j E(Z_j^* | H_0)}{\sqrt{\sum_j W_j^2 Var(Z_j^* | H_0)}}$$

The Balancing Critical Value

There are four key elements of the statistical testing process:

- the null hypothesis, H_0, that parity exists between ILEC and CLEC services
- the alternative hypothesis, H_a, that the ILEC is giving better service to its own customers
- the Truncated Z test statistic, Z^T, and
- a critical value, c

The decision rule2 is

- If Z^T ~~$< c$ then~~ $Z^T < c$ then accept H_a.
- If Z^T ~~$\geq c$ then~~ Z^T □ c then accept H_0.

There are two types of error possible when using such a decision rule:

- **Type I Error**: Deciding favoritism exists when there is, in fact, no favoritism.
- **Type II Error**: Deciding parity exists when there is, in fact, favoritism.

The probabilities of each type of each are:

- **Type I ~~Error:~~** ~~$\alpha = P(Z$~~

~~**Type II**~~

- **Type II Error**:

$\beta = P(Z \geq c | H)$

We want a balancing critical value, c_B, so that $\alpha = \beta$.

It can be shown that.

~~**Trim the ILEC observations to the largest CLEC value from all CLEC observations in the month under consideration.**~~

~~That is, no CLEC values are removed; all ILEC observations greater than the largest CLEC observation are trimmed.~~

2 This decision rule assumes that a negative test statistic indicates poor service for the CLEC ~~customer.~~ customer. If the opposite is true, then reverse the decision rule.

$$\cancel{V(\mu,\sigma) = (\mu^2+\sigma^2)\Phi(\tfrac{-\mu}{\sigma}) - \mu\sigma\phi(\tfrac{-\mu}{\sigma}) - M}$$

$$c = \frac{\sum_j W_j M(m_j, se_j) - \sum_j W_j \frac{-1}{\sqrt{2\pi}}}{\sqrt{\sum_j W_j^2 V(m_j, se_j)} + \sqrt{\sum_j W_j^2 \left(\frac{1}{2} - \frac{1}{2\pi}\right)}}$$

~~where~~

$$\cancel{M(\mu,\sigma) = \mu\,\Phi(\tfrac{-\mu}{\sigma}) - \sigma\phi(\tfrac{-\mu}{\sigma})}$$

where

$$M(\mu,\sigma) = \mu\Phi(\tfrac{-\mu}{\sigma}) - \sigma$$

$$V(\mu,\sigma) = (\mu \;+\sigma\;)\Phi(\tfrac{-\mu}{\sigma}) - \mu\sigma\,\phi(\tfrac{-\mu}{\sigma}) - M($$

$\Phi(\cdot)\Phi(\square)$ is the cumulative standard normal distribution function, and $\phi(\cdot)\phi(\square)$ is the standard normal density function.

This formula assumes that Z_j is approximately normally distributed within cell j. When the cell sample sizes, n_{1j} and n_{2j}, are small this may not be true. It is possible to determine the cell mean and variance under the null hypothesis when the cell sample sizes are small. It is much more difficult to determine these values under the alternative hypothesis. Since the cell weight, W_j will also be small (see calculate weights section above) for a cell with small volume, the cell mean and variance will not contribute much to the weighted sum. Therefore, the above formula provides a reasonable approximation to the balancing critical value.

The values of m_j and se_j will depend on the type of performance measure.

Mean Measure

For mean measures, one is concerned with two parameters in each cell, namely, the mean and variance. A possible lack of parity may be due to a difference in cell means, and/or a difference in cell variances. One possible set of hypotheses that capture this notion, and take into account the assumption that transaction are identically distributed within cells is:

$$H_0: \mu_{1j} = \mu_{2j},\ \sigma_{1j}^2 = \sigma_{2j}^2$$

$$H_a: \mu_{2j} = \mu_{1j} + \delta_j \cdot \sigma_{1j},\ \sigma_{2j}^2 = \lambda_j \cdot \sigma_{1j}^2 \quad \delta_j \cancel{> 0, \lambda_j \geq 1 \text{ and } j = 1,\ldots,L.}$$

Under this form of alternative hypothesis, the cell test statistic Z_j has mean and standard error given by

$$m = \frac{-}{\sqrt{-}}$$

$$m_j = \frac{-\delta_j}{\sqrt{\frac{1}{n_{1j}} + \frac{1}{n_{2j}}}}, \text{ and}$$

$$se = \sqrt{\frac{\lambda\, n}{n \cdot}}$$

Proportion Measure

For a proportion measure there is only one parameter of interest in each cell, the proportion of transaction possessing an attribute of interest. A possible lack of parity may be due to a difference in cell proportions. A set of hypotheses that take into account the assumption that transaction are identically distributed within cells while allowing for an analytically tractable solution is:

$$H_0: \frac{p_{2j}(1-p_{1j})}{(1-p_{2j})p_{1j}} = 1$$

$$H_a: \frac{p_{2j}(1-p_{1j})}{(1-p_{2j})p_{1j}} = \psi_j \qquad \psi_j > 1 \text{ a}$$

$$\frac{p\ (1-p\ )}{(1-p\ )p} = 1$$

H_a:

$$\frac{p\ (1-p\ )}{(1-p\ )p} = \psi$$

These hypotheses are based on the "odds ratio." If the transaction attribute of interest is a missed trouble repair, then an interpretation of the alternative hypothesis is that a CLEC trouble repair appointment is ψ_j times more likely to be missed than an ILEC trouble.

Under this form of alternative hypothesis, the within cell asymptotic mean and variance of a_{1j} are given by3

$$E(a\ ) = n\,\pi$$

$$\mathrm{var}(a\ ) = \frac{n}{\frac{}{} + \frac{}{} + \frac{}{}}$$

where

3 Stevens, W. L. (1951) Mean and Variance of an entry in a Contingency Table. *Biometrica*, 38, 468- 470.

$$\pi = f \left(n + f + f - f\right)$$
$$\pi = f \left(-n - f + f + f\right)$$
$$\pi = f \left(-n + f - f + f\right)$$
$$\pi = f \; n \; \overline{\psi} - 1 - f - f - f$$
$$f = \frac{1}{2n \; \overline{\psi} - 1}$$
$$f = n n \; \overline{\psi} - 1$$
$$f = n a \; \overline{\psi} - 1$$
$$f = \sqrt{n \left[4n \left(n - a\right) \overline{\psi} - 1 + n + \left(a - n\right) \overline{\psi} - 1\right.}$$

Recall that the cell test statistic is given by

$$Z_j = \frac{n_j a_{1j} - n_{1j} a}{\sqrt{\frac{n_{1j} n_{2j} a_j (n_j -}{n_j - 1}}}$$

Using the equations above, we see that Z_j has mean and standard error given by

$$m_j = \frac{n_j^2 \pi_j^{(1)} - n_{1j} a_j}{\sqrt{\frac{n_{1j} n_{2j} a_j (n_j - a_j)}{n_j - 1}}}, \text{ and}$$

$$se_j = \sqrt{\frac{}{n_{1j} n_{2j} a_j (n_j -}}$$

and

$$se = \sqrt{\frac{n (n - 1)}{n \; n \; a \; (n - a) \; \frac{}{} + \frac{}{} + \frac{}{}}}$$

Rate Measure

A rate measure also has only one parameter of interest in each cell, the rate at which a phenomenon is observed relative to a base unit, e.g. the number of troubles per available line. A possible lack of parity may be due to a difference in cell rates. A set of hypotheses that take into account the assumption that transaction are identically distributed within cells is:

H_0: $r_{1j} = r_{2j}$

H_a: $r_{2j} = \varepsilon_j r_{1j}$ ~~$\varepsilon_j > 1$ and $j = 1, \ldots, L$.~~ $\varepsilon_j > 1$ and $j = 1, \ldots, L$.

Given the total number of ILEC and CLEC transactions in a cell, n_j, and the number of base elements, b_{1j} and b_{2j}, the number of ILEC transaction, n_{1j}, has a binomial distribution from n_j trials and a probability of

$$q_j^* = \frac{r_{1j} b_{1j}}{r_{1j} b_{1j} + r_{2j} b_{2j}}.$$

Therefore, the mean and variance of n_{1j}, are given by

$$E(n \ ) = n q$$
$$var(n \ ) = n q (1$$

Under the null hypothesis

$$q_j^* = q_j =$$

;

but under the alternative hypothesis

$$q_j^* = q_j^a = \frac{b_1}{b_{1j} +}$$

.

Recall that the cell test statistic is given by

$$Z_j = \frac{n_{1j} - n_j}{\sqrt{n_j q_j (1 -}}$$

.

Using the relationships above, we see that Z_j has mean and standard error given by

$$m_j = \frac{n_j\left(q_j^a - q_j\right)}{\sqrt{n_j q_j (1 - q_j)}} = (1 - \varepsilon_j)\frac{\sqrt{n_j b_{1j} b_{2j}}}{b_{1j} + \varepsilon_j b_{2j}}, \text{ and}$$

$$se_j = \sqrt{\frac{q_j^a(1 - q_j^a)}{q_j(1 - q_j)}} = \sqrt{\varepsilon_j}\frac{b_j}{b_{1j} + \varepsilon_j b_{2j}}.$$

$$m \ = \frac{n \ (q \ - q \ )}{\sqrt{n q (1 - q \ )}} = (1 - \varepsilon \ )\frac{\sqrt{n}}{b \ +}$$

$$se = \sqrt{\frac{q(1-q)}{q(1-q)}} = \sqrt{\varepsilon} \frac{1}{b +}$$

and

Ratio Measure

As with mean measures, one is concerned with two parameters in each cell, the mean and variance, when testing for parity of ratio measures. As long as sample sizes are large, as in the case of billing accuracy, the same method for finding m_j and se_j that is used for mean measures can be used for ratio measures.

Determining the Parameters of the Alternative Hypothesis

In this ~~appendix~~section we have indexed the alternative hypothesis of mean measures by two sets of parameters, ~~λ_j and δ_j.~~ λ_j and δ_j. Proportion and rate measures have been indexed by one set of parameters each, ψ_j and ε_j respectively. A major difficulty with this approach is that more than one alternative will be of interest; for example we may consider one alternative in which all the δ_j are set to a common non-zero value, and another set of alternatives in each of which just one δ_j is non-zero, while all the rest are zero. There are very many other possibilities. Each possibility leads to a single value for the balancing critical value; and each possible critical value corresponds to many sets of alternative hypotheses, for each of which it constitutes the correct balancing value.

The formulas we have presented can be used to evaluate the impact of different choices of the overall critical value. For each putative choice, we can evaluate the set of alternatives for which this is the correct balancing value. While statistical science can be used to evaluate the impact of different choices of these parameters, there is not much that an appeal to statistical principles can offer in directing specific choices. Specific choices are best left to telephony experts. Still, it is possible to comment on some aspects of these choices:

Parameter Choices for ~~λ_j.~~ λ_j – The set of parameters $\lambda_j\lambda_j$ index alternatives to the null hypothesis that arise because there might be greater unpredictability or variability in the delivery of service to a CLEC customer over that which would be achieved for an otherwise comparable ILEC customer. While concerns about differences in the variability of service are important, it turns out that the truncated Z testing which is being recommended here is relatively insensitive to all but very large values of the ~~λ_j.~~ λ_j. Put another way, reasonable differences in the values chosen here could make very little difference in the balancing points chosen.

Parameter Choices for ~~δ_j.~~ δ_j – The set of parameters $\delta_j\delta_j$ are much more important in the choice of the balancing point than was true for the ~~λ_j.~~ λ_j. The reason for this is that they directly index differences in average service. The truncated Z test is very sensitive to any such differences; hence, even small disagreements among experts in the choice of the $\delta_j\delta_j$ could be very important. Sample size matters here too. For example, setting all the $\delta_j\delta_j$ to a single ~~value – $\delta_j = \delta$ –~~ value – $\delta_j = \delta$□ might be fine for tests across individual CLECs where currently in Georgia the CLEC customer bases are not too different. Using the same value of $\delta\delta$ for the overall state testing does not seem sensible. At the state level we are aggregating over CLECs, so using the same $\delta\delta$ as for an individual CLEC would be saying that a ~~"meaningful"~~ "meaningful" degree of disparity is one where the violation is the same ~~(δ)~~(δ) for each CLEC. But the detection of disparity for any component CLEC is important, so the relevant ~~"overall" δ~~ "overall" δ should be smaller.

--Parameter Choices for ψ_j *or* ~~ε_j~~ ε_j – The set of parameters ψ_j or ε_j are also important in the choice of the balancing point for tests of their respective measures. The reason for this is that they directly index increases in the proportion or rate of service performance. The truncated Z test is sensitive to such increases; but not as sensitive as the case of ~~δ~~ δ for mean measures. Sample size matters here too. As with mean measures, using the same value of ψ or ε for the overall state testing does not seem sensible.

$$\delta = 2 \cdot \arcsin(\sqrt{\hat{p}_2}) - 2 \cdot \arcsin($$

$$\delta = 2\sqrt{\hat{r}_2} - 2\sqrt{\hat{r}_1}$$

The three parameters are related however. If a decision is made on the value of ~~δ,~~δ, it is possible to determine equivalent values of ψ and ε. The following equations, in conjunction with the definitions of ψ and ε, show the relationship with delta.

The bottom line here is that beyond a few general considerations, like those given above, a principled approach to the choice of the alternative hypotheses to guard against must come from elsewhere.

Decision Process

Once Z^T has been calculated, it is compared to the balancing critical value to determine if the ILEC is favoring its own customers over a CLEC's customers.

This critical value changes as the ILEC and CLEC transaction volume change. One way to make this transparent to the decision-maker, is to report the difference between the test statistic and the critical value, $diff = Z^T - c_B$. If favoritism is concluded when $Z^T < c_B$, then the $diff < 0$ indicates favoritism.

This makes it very easy to determine favoritism: a positive *diff* suggests no favoritism, and a negative *diff* suggests favoritism.

~~Corrections~~

~~LPSC "Statistical Techniques for the Analysis and Comparison of Performance Measure Data".~~

~~Appendix A, page A-5~~

$$T_j = t_j + \frac{g}{6}\left(\frac{n_{1j}+2n_{2j}}{\sqrt{n_{1j}\, n_{2j}(n_{1j}+n_{2j})}}\right)\left(t_j^2 + \frac{n_{2j}-n_{1j}}{n_{1j}+2n_{2j}}\right)$$

~~Appendix C, page C-8, rate measures section for balancing critical value.~~

$$m_j = \frac{n_j\left(q_j^a - q_j\right)}{\sqrt{n_j q_j(1-q_j)}} = (1-\varepsilon_j)\frac{\sqrt{n_j b_{1j} b_{2j}}}{b_{1j}+\varepsilon_j b_{2j}}$$

~~APPENDIX E~~

: <u>BST SEEM Remedy Calculation Procedures</u>

BST SEEM Remedy Procedure

~~BST SEEM REMEDY PROCEDURE~~

~~TIER-1 CALCULATION FOR RETAIL ANALOGUES:~~

1. Tier-1 Calculation For Retail Analogues

2. Calculate the overall test statistic for each CLEC; z^TCLEC-1 (Per Statistical Methodology ~~discussed~~ by Dr. Mulrow)
~~2.~~3. Calculate the balancing critical value ($^cB_{CLEC-1}$) that is associated with the alternative hypothesis (for fixed parameters ~~δ,Ψ, or ε)~~δ,Ψ, or ε)
4. If the overall test statistic is equal to or above the balancing critical value, stop here. That is, if $^cB_{CLEC-1} < z^T$CLEC-1, stop here. Otherwise, go to step 4.
5. Calculate the Parity Gap by subtracting the value of step 2 from that of step 1. ABS (z^TCLEC-1 - $^cB_{CLEC-1}$)
6. Calculate the Volume Proportion using a linear distribution with slope of ¼. This can be accomplished by taking the absolute value of the Parity Gap from step 4 divided by 4; ABS ((z^TCLEC-1 - $^cB_{CLEC-1}$) / 4). All parity gaps equal or greater to 4 will result in a volume proportion of 100%.
~~6.~~7. Calculate the Affected Volume by multiplying the Volume Proportion from step 5 by the Total Impacted CLEC-$_1$ Volume (I_c) in the negatively affected cell; where the cell value is negative.
8. Calculate the payment to CLEC-1 by multiplying the result of step 6 by the appropriate dollar amount from the fee schedule.
9. Then, CLEC-1 payment = Affected Volume$_{CLEC1}$ * $$-from Fee Schedule

Example: CLEC-1 Missed Installation Appointments (MIA) for Resale POTS~~.~~

Note – the statistical results are only illustrative. They are not a result of a statistical test of this data.

	~~n_I~~	~~N_C~~	~~I_c~~	~~MIA_I~~	~~MIA_C~~	~~$z^T{}_{CLEC-1}$~~	~~C_B~~	~~Parity Gap~~	~~Volume Proportion~~	~~Affected Volume~~
	nI	**NC**	**Ic**	**MIAI**	**MIAC**	**zTCLEC-1**	**CB**	**Parity Gap**	**Volume Proportion**	**Affected Volume**
State	50000	600	96	9%	16%	-1.92	-0.21	1.71	0.4275	
Cell						zCLEC-1				
1		150	17	0.091	0.113	-1.994				8
2		75	8	0.176	0.107	0.734				
3		10	4	0.128	0.400	-2.619				2
4		50	17	0.158	0.340	-2.878				8
5		15	2	0.245	0.133	1.345				
6		200	26	0.156	0.130	0.021				
7		30	7	0.166	0.233	-0.600				3
8		20	3	0.106	0.150	-0.065				2
9		40	9	0.193	0.225	-0.918				4
10		10	3	0.160	0.300	-0.660				2
										29

where n_I = ILEC observations and n_C = CLEC-1 observations

Payout for CLEC-1 is (29 units) * ($100/unit) = $2,900

Example: CLEC-1 Order Completion Interval (OCI) for Resale POTS

	~~n_I~~	~~n_C~~	~~I_c~~	~~OCI_I~~	~~OCI_C~~	~~z^T_{CLEC-1}~~	~~C_B~~	~~Parity Gap~~	~~Volume Proportion~~	~~Affected Volume~~

	n I	**nC**	**Ic**	**OCII**	**OCIC**	**zTCLEC-1**	**CB**	**Parity Gap**	**Volume Proportion**	**Affected Volume**
State	50000	600	600	5days	7days	-1.92	-0.21	1.71	0.4275	
Cell						zCLEC-1				
1		150	150	5	7	-1.994				64
2		75	75	5	4	0.734				
3		10	10	2	3.8	-2.619				4
4		50	50	5	7	-2.878				21
5		15	15	4	2.6	1.345				
6		200	200	3.8	2.7	0.021				
7		30	30	6	7.2	-0.600				13
8		20	20	5.5	6	-0.065				9
9		40	40	8	10	-0.918				17
10		10	10	6	7.3	-0.660				4

133

where n_I = ILEC observations and n_C = CLEC-1 observations

Payout for CLEC-1 is (133 units) * ($100/unit) = $13,300

~~**TIER-2 CALCULATION for RETAIL ANALOGUES:**~~
1.

Tier-2 Calculation For Retail Analogues

1. Tier-2 is triggered by three consecutive monthly failures of any Tier 2 Remedy Plan sub-metric.
2. Therefore, calculate monthly statistical results and affected volumes as outlined in steps 2 through 6 for the CLEC Aggregate performance. Determine average monthly affected volume for the rolling 3-month period.
3. Calculate the payment to State Designated Agency by multiplying average monthly volume by the appropriate dollar amount from the Tier-2 fee schedule.
4. Therefore, State Designated Agency payment = Average monthly volume * $$-from Fee Schedule

Example: CLEC-A Missed Installation Appointments (MIA) for Resale POTS

~~State~~	~~n_I~~	~~n_C~~	~~I_c~~	~~MIA_I~~	~~MIA_C~~	~~z^T_{CLEC-A}~~	~~C_B~~	~~Parity Gap~~	~~Volume Proportion~~	~~Affected Volume~~
State	**nI**	**nC**	**Ic**	**MIAI**	**MIAC**	**zTCLEC-A**	**CB**	**Parity Gap**	**Volume Proportion**	**Affected Volume**
Month 1	180000	2100	336	9%	16%	-1.92	-0.21	1.71	0.4275	
Cell						zCLEC-A				
1		500	56	0.091	0.112	-1.994				24
2		300	30	0.176	0.100	0.734				
3		80	27	0.128	0.338	-2.619				12
4		205	60	0.158	0.293	-2.878				26
5		45	4	0.245	0.089	1.345				
6		605	79	0.156	0.131	0.021				
7		80	19	0.166	0.238	-0.600				9
8		40	6	0.106	0.150	-0.065				3
9		165	36	0.193	0.218	-0.918				16
10		80	19	0.160	0.238	-0.660				9
										99

where n_I = ILEC observations and n_C = CLEC-A observations

Assume Months 2 and 3 have the same affected volumes. Payout 99 units * $300/unit = $29,700.

If the above example represented performance for each of months 1 through 3, then

Example: CLEC-A Missed Installation Appointments for 1Q00

State	Miss	Remedy Dollars
Month 1	X	$29,700
Month 2	X	$29,700
Month 3	X	$29,700
1Q00		$89,100

~~TIER-1 CALCULATION FOR BENCHMARKS~~

1.

Tier-1 Calculation For Benchmarks

1. For each CLEC, with five or more observations, calculate monthly performance results for the State.

2. CLECs having observations (sample sizes) between 5 and 30 will use Table I below. The only exception will be for Collocation Percent Missed Due Dates.

~~Table I~~ ~~Small Sample Size Table~~

~~(95%~~Table I - Small Sample Size Table (95% Confidence)

Sample Size	Equivalent 90% Benchmark	Equivalent 95% Benchmark	Sample Size	Equivalent 90% Benchmark	Equivalent 95% Benchmark
~~5~~	~~60.00%~~	~~80.00%~~	~~16~~	~~75.00%~~	~~87.50%~~
5	60.00%	80.00%	18	77.78%	83.33%
~~6~~	~~66.67%~~	~~83.33%~~	~~17~~	~~76.47%~~	~~82.35%~~
6	66.67%	83.33%	19	78.95%	84.21%
~~7~~	~~71.43%~~	~~85.71%~~	~~18~~	~~77.78%~~	~~83.33%~~
7	71.43%	85.71%	20	80.00%	85.00%
~~8~~	~~75.00%~~	~~75.00%~~	~~19~~	~~78.95%~~	~~84.21%~~
8	75.00%	75.00%	21	76.19%	85.71%
~~9~~	~~66.67%~~	~~77.78%~~	~~20~~	~~80.00%~~	~~85.00%~~
9	66.67%	77.78%	22	77.27%	86.36%
~~10~~	~~70.00%~~	~~80.00%~~	~~21~~	~~76.19%~~	~~85.71%~~
10	70.00%	80.00%	23	78.26%	86.96%
~~11~~	~~72.73%~~	~~81.82%~~	~~22~~	~~77.27%~~	~~86.36%~~
11	72.73%	81.82%	24	79.17%	87.50%
~~12~~	~~75.00%~~	~~83.33%~~	~~23~~	~~78.26%~~	~~86.96%~~
12	75.00%	83.33%	25	80.00%	88.00%
~~13~~	~~76.92%~~	~~84.62%~~	~~24~~	~~79.17%~~	~~87.50%~~
13	76.92%	84.62%	26	80.77%	88.46%
~~14~~	~~78.57%~~	~~85.71%~~	~~25~~	~~80.00%~~	~~88.00%~~
14	78.57%	85.71%	27	81.48%	88.89%
~~15~~	~~73.33%~~	~~86.67%~~	~~26~~	~~80.77%~~	~~88.46%~~
			~~27~~	~~81.48%~~	~~88.89%~~
			~~28~~	~~78.57%~~	~~89.29%~~
15	73.33%	86.67%	28	78.57%	89.29%
			~~29~~	~~79.31%~~	~~86.21%~~
16	75.00%	87.50%	29	79.31%	86.21%
			~~30~~	~~80.00%~~	~~86.67%~~
17	76.47%	82.35%	30	80.00%	86.67%

~~3.~~

1. If the percentage (or equivalent percentage for small samples) meets the benchmark standard, stop here. Otherwise, go to step 4.